CONJUGATE HEAT TRANSFER AND THERMOSTRESSED STATE OF THE WALL IN STEADY FLOW IN A TRIANGULAR PIPE

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UDC 536.24:539.3

Consideration is given to the problem of joint analysis for the parameters of conjugate heat transfer and the thermostressed state of the wall in steady laminar flow in a thick-walled triangular pipe. The solution is obtained by successive use of a finite-element method (FEM) and a finite-difference method (FDM). Analysis is given of the results of calculations at different values of similarity criteria.

Intensification of heat transfer in pipes and channels of power plants for various purposes is accompanied by an increase of operating temperatures, at which the construction elements washed by heat transfer agents function. These changes in turn inevitably influence the thermostrength characteristics of the equipment. Therefore, when evaluating the efficiency of technical solutions aimed at increasing heat transfer through the surfaces washed by the heat-transfer agents, the analysis of conjugate heat transfer in the liquid-wall system is to be accompanied by a calculation of the thermostressed state of the construction.

The present work is devoted to the joint analysis of thermal and deformation processes, occuring in a thick-walled triangular pipe (the triangle is isosceles) on its initial thermal portion, the most interesting from the practical point of view. It is assumed that the flow in the pipe is laminar and hydrodynamically stabilized. The temperatures in the inlet section and on the outer surface of the pipe are assigned and the thermophysical properties of the liquid and the wall material are constant. In calculating thermostresses it is assumed that in the inlet section the pipe is fixed and its lateral surface is free from mechanical loads.

With due regard to the foregoing, a mathematical model, describing stationary conjugate heat transfer in the channel-wall system (Fig. 1) and the corresponding thermostressed state of the pipe, will be presented in the Cartesian coordinate system in dimensionless variables by the following equations [1, 2]:

$$L\,\mathrm{Pe}\,W_{z_i}\frac{\partial \Theta_i}{\partial Z}=K_{\lambda_i}\left(\frac{\partial^2\Theta_i}{\partial X^2}+\frac{\partial^2\Theta_i}{\partial Y^2}+\frac{\partial^2\Theta_i}{\partial Z^2}L^2\right); \tag{1}$$

$$(1-2\nu)\,\Delta'\mathbf{U}+\mathrm{grad}'\,\mathrm{div}'\,\mathbf{U}=2(1+\nu)\,\mathrm{grad}'\,\Theta_2. \tag{2}$$

Here i = 1 is the liquid, i = 2 is the wall;

$$\Theta_i=\frac{T_i-T_{\mathrm{in}}}{T_w-T_{\mathrm{in}}};\quad K_{\lambda_i}=\lambda_i/\lambda_l;\quad X=x/h;\quad Y=y/h;\quad Z=z/l;$$

$$L=h/l;\quad W_Z=w_z/\bar{w};\quad \mathrm{Pe}=\bar{w}h/a_l;\quad \mathbf{U}=\frac{\mathbf{u}}{\alpha_{\mathrm{p}}(T_w-T_{\mathrm{in}})\,h},$$

$$\mathbf{u}=\{u_x,\ u_y,\ u_z\};\quad \Delta'=\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}+L^2\frac{\partial^2}{\partial Z^2};$$

$$\mathrm{grad}'=\mathbf{i}\frac{\partial}{\partial X}+\mathbf{j}\frac{\partial}{\partial Y}+\mathbf{k}L\frac{\partial}{\partial Z};$$

$$\mathrm{div}'=\frac{\partial}{\partial X}+\frac{\partial}{\partial Y}+L\frac{\partial}{\partial Z};\quad K_\delta=\frac{\delta}{h};\quad W_{Z_2}=0;$$

Dnepropetrovsk State University. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 64, No. 1, pp. 107-112, January, 1993. Original article submitted October 22, 1991.

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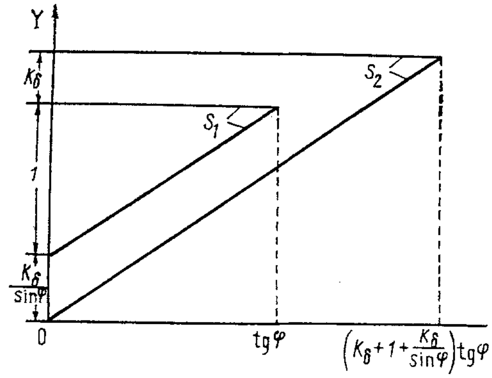

Fig. 1

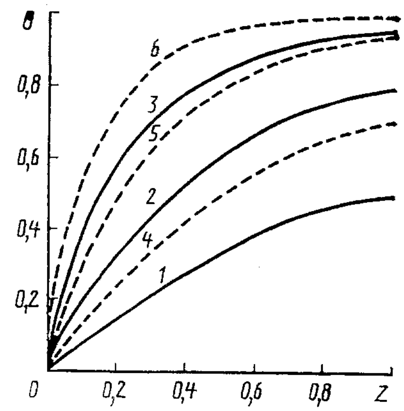

Fig. 2

Fig. 1. Calculated region.

Fig. 2. Distribution of the bulk temperature along the pipe axis for Pe = 50; K_δ = 0.3: 1) K_λ = 0.25; 2) 1; 3) 10 at φ = 60°; 4-6) the same, at φ = 30°

$$\Theta_i|_{Z=0} = U_X|_{Z=0} = U_Y|_{Z=0} = U_Z|_{Z=0} = 0; \tag{3}$$

$$\left.\frac{\partial\Theta_i}{\partial Z}\right|_{Z=1} = U_Z|_{Z=1} = \sigma_{XZ}|_{Z=1} = \sigma_{YZ}|_{Z=1} = 0; \tag{4}$$

$$\Theta_2|_{S_2;0<Z<1} = 1; \tag{5}$$

$$\begin{gathered}\sigma_{XX}n_X + \sigma_{XY}n_Y|_{S_1,S_2;\ 0<Z<1} = 0,\\ \sigma_{XY}n_X + \sigma_{YY}n_Y|_{S_1,S_2;\ 0<Z<1} = 0;\end{gathered} \tag{6}$$

$$\begin{gathered}U_X = \sigma_{XY} = 0 \quad \text{for} \quad X = 0;\ 0 \leqslant Y \leqslant \frac{K_\delta}{\sin\varphi},\\ 1 \leqslant Y \leqslant 1 + K_\delta,\ \ 0 < Z < 1;\end{gathered} \tag{7}$$

$$\Theta_1|_{S_1} = \Theta_2|_{S_1};\ \left.\frac{\partial\Theta_1}{\partial N}\right|_{S_1} = K_\lambda\left.\frac{\partial\Theta_2}{\partial N}\right|_{S_1} \quad \text{for} \quad 0 < Z < 1, \tag{8}$$

where $K_\lambda = \lambda_2/\lambda_1$.

The parameter L in Eqs. (1) and (2) is taken equal to 1/Pe, which corresponds to the distance from the pipe outlet at which stabilization of the liquid and wall temperatures, and consequently of thermostresses in the axial direction, will take place. Such a choice is made on the basis of estimates for the sizes of the initial thermal portions obtained in solving similar problems of convective heat transfer in a nonconjugate statement [3].

On discretization of the channel cross-sectional area (Fig. 1) into finite elements, let us choose a linear basis for approximating the temperature and displacement vector components, whose employment leads to the known calculated relations for each element [4]

$$\begin{gathered}\Theta_i^{(e)} = [N^{(e)}]\{\Theta_i\};\quad U_X^{(e)} = [N^{(e)}]\{U_X\};\\ U_Y^{(e)} = [N^{(e)}]\{U_Y\};\quad U_Z^{(e)} = [N^{(e)}]\{U_Z\}.\end{gathered} \tag{9}$$

Realizing the finite-element method using Galerkin's procedure and performing the assembly by elements over the entire calculated region, to determine node values of temperature and displacement vector components we will obtain systems of differential equations of the form

$$[A]\frac{d^2\{\Theta\}}{dZ^2} + [B]\frac{d\{\Theta\}}{dZ} + [C]\{\Theta\} + \{D\} = 0; \tag{10}$$

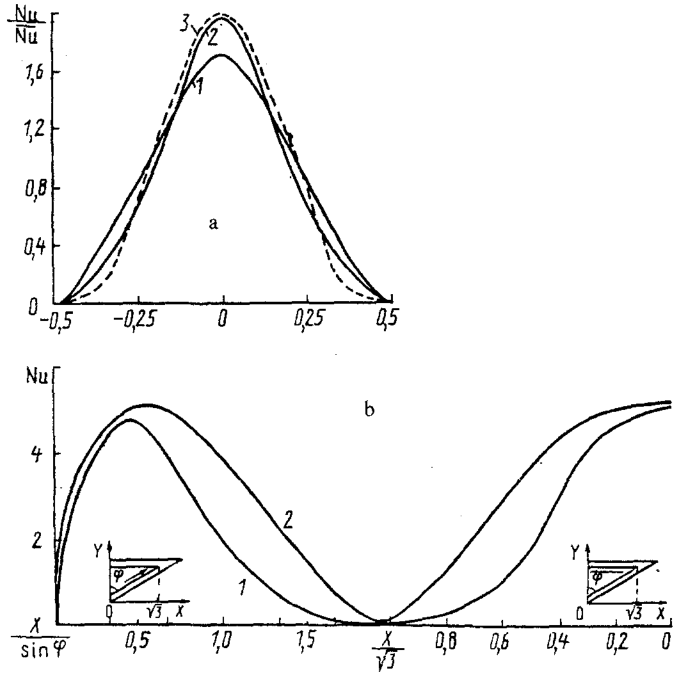

Fig. 3. Distribution of local values of Nu ($Nu = \alpha d_e/\lambda_\ell$) along the perimeter of the triangular channel at φ = 30 (a) and 60° (b) for Pe = 5; K_δ = 0.3; K_λ = 1: 1) Z = 0.1; 2) 0.5; 3) $Nu_\infty/\overline{Nu}]_\infty$ ratio from [3].

$$[A_1]\frac{d^2\{U\}}{dZ^2}+[B_1]\frac{d\{U\}}{dZ}+[C_1]\{U\}+\{D_1\}=0, \tag{11}$$

where [A], [B], [C], $[A_1]$, $[B_1]$, and $[C_1]$ are band matrices; $\{\Theta\}$, $\{U\}$ are the vectors of the node unknowns; $\{D\}$, $\{D_1\}$ are the vectors taking the source terms of the system (1), (2) and the boundary conditions into account.

The systems of equations (10) and (11) are solved by the finite-difference method with the use of Sauliev's schema [5].

The above method was originally developed in [6] for calculating the processes of heat transfer in pipes and channels. Its utilization in the present work to solve the system of thermoelasticity equations has required the introduction of a certain modification into this method. The need to improve the method is explained by a difference in the structure of the matrix-coefficients [B] and $[B_1]$ of the convective terms in the matrix equations (10) and (11). In the system (10) the matrix [B] is of constant sign because the flow of the heat-transfer agent in the pipe does not change its direction (sign W_Z = const). The structure of the thermoelasticity equations (the presence of the combined derivatives $\partial^2/\partial X\partial Z$, $\partial^2/\partial Y\partial Z$) is such that the matrix [B] in the system (11) is not of constant sign as is shown by numerical analysis. This requires the representation of the matrix $[B_1]$ as a sum of constant-sign matrices $[B_1] = [B_1^+] + [B_1^-]$ and the approximation of the convective term in the system (11) by one-sided differences, oriented against the perturbation direction [7]

$$[B_1]\frac{d\{U\}}{dZ}\approx[B_1^+]\frac{\{U\}_j-\{U\}_{j-1}}{h_z}+[B_1^-]\frac{\{U\}_{j+1}-\{U\}_j}{h_z}. \tag{12}$$

Such a representation of the convective term in the system (11) ensures the stability of Sauliev's iteration schema [5]. The idea to use approximation (12) is based on the fact that this procedure has shown a good performance in solving the problems of hydrogasdynamics by finite-difference methods.

Using the above method, a complex analysis is performed of the stationary conjugate heat transfer and the thermostressed state of the wall in a laminar, stabilized flow of a liquid in a thick-walled pipe, whose cross section is an isosceles triangle. Most attention is paid to elucidating the influence of similarity criteria, determining the conjugate heat transfer in the liquid-wall system (K_λ, K_δ, Pe), on the thermostressed state of the pipe.

Some results of the calculations are given in Figs. 2-4.

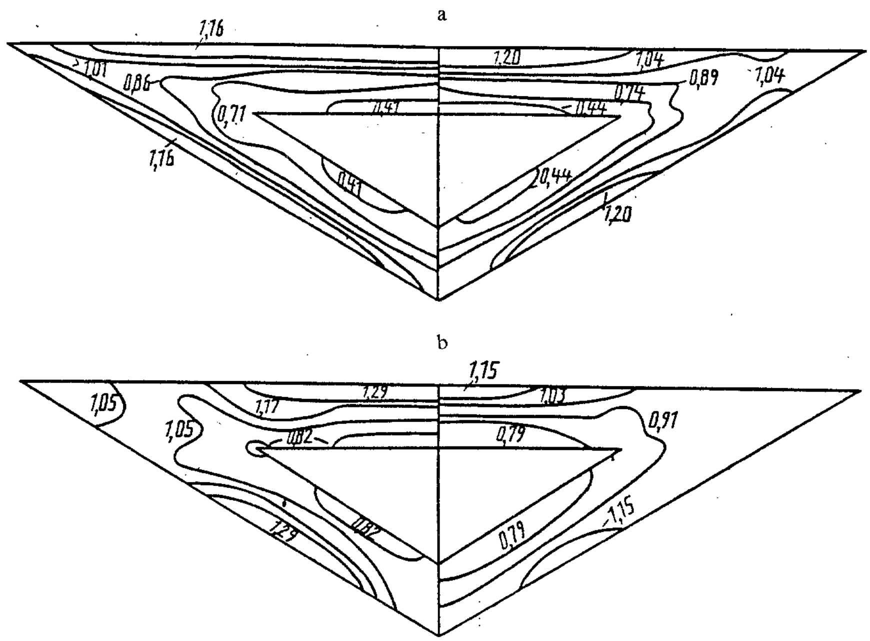

Fig. 4. Distribution of the intensity of stresses σ_i in the cross section of the triangular pipe at $K_\lambda = 0.25$; $K_\delta = 0.3$; Pe = 5 (the left half); Pe = 50 (the right half); a) Z = 0.1; b) 0.5.

It can be seen from Fig. 2 that for a thick-walled pipe the parameter determining the dynamics of heating of the liquid in the axial direction is the criterion K_λ. Comparison of the results for pipes with equilateral ($\varphi = 30°$) and obtuse ($\varphi = 60°$) triangles in cross section shows that in the first case an axial variation of the bulk temperature is characterized by a larger velocity as well as by higher asymptotic values. This is explained by the fact that when the height of both triangles is fixed (dimensionless) (equal to 1, since the dimensional height is taken as the characteristic dimension in nondimensionalization) and values of the Pe number are the same, there is a smaller flow rate of the liquid through the pipe with the equilateral triangle in the cross section.

The variation in the heat transfer between the liquid and the wall along the perimeter of the pipe cross section can be judged from the results given in Fig. 3. From Fig. 3a (curve 2) it is seen that it is already on half the length of the considered inlet section of the pipe that the distribution of local values of Nu in the axial direction is practically stabilized, which is confirmed by its good agreement with the analogous asymptotic distribution, obtained by solving the similar problem of heat transfer in a nonconjugate statement (see [3]). The largest discrepancies are observed in the vicinity of the angular points and are explained by errors of numerical differentiation in the procedure of FEM when calculating the local values of Nu.

Characteristic of a pipe having an obtuse triangle in plan are the following features of heat transfer along the perimeter of the cross section (Fig. 3b): 1) the maximum of local values of Nu on the triangle side-base somewhat exceeds the maximum on the lateral side; 2) unlike the pipe with an equilateral triangle (Fig. 3a) the maximum point on the lateral side is displaced to the vertex by an obtuse angle. The first of the indicated features is explained by the fact that the symmetry axis of the pipe side, which serves as a base of the triangle, is most distant from the angular zones, in which the stagnation of the stream occurs. The second feature is a consequence of the fact that the stagnation zone of the stream in the vicinity of the obtuse angle is smaller than the analogous zones adjacent to the acute angles at its base. As the local maximums of the local values of $\overline{Nu}$ recede from the pipe inlet, they become more sharply defined (Fig. 3b, curve 2).

Figure 4 shows the pictures of isolines for the intensity of stresses σ_i for two axial sections of the pipe at various Pe, which enable one to make a certain integral estimate of the thermostressed state of the pipe. The largest temperature stresses are observed on the outer surface of the pipe, i.e., where its temperature is maximal. Noteworthy is the indissoluble relation between the configuration of the maximal thermostress regions and the character of distribution of local values of Nu along the channel perimeter (Fig. 3b). These regions have the largest depth in the wall thickness at those points of the perimeter where the local values attain the local maximums. This is explained by the fact that it is the temperature gradient across the pipe wall thickness that has the maximal values in these zones. With distance from the pipe inlet (Fig. 4b) the length of the maximal thermostress zones reduces, which corresponds to the localization of the sections of maximums for the local values of Nu (Fig. 3b, curve 2). A comparison of Fig. 4a and 4b shows that at small values of Pe (Pe = 5) thermostresses grow as the distance from the pipe inlet increases; by increasing Pe to the value Pe = 50 the opposite picture is observed. Let us analyze these differences. With Pe = 5 in the cross section Z = 0.1 the dominant influence on the thermostress level is exerted by the temperature gradient across the wall thickness, which regularly decreases in the axial direction as the liquid is heated. However, this drop of the temperature gradient has an excess compensated by heating the pipe across the wall thickness in the axial direction, which ultimately causes the growth of thermostresses. With Pe = 50 the heat transfer between the liquid and the pipe is intensified; this leads to a more abrupt drop of the temperature gradient across the wall thickness, which is no longer compensated by the overall heating of the pipe. As a result, the general level of thermostresses in the pipe is reduced.

As a whole, the calculations performed show that a variation in any parameter, affecting conjugate heat transfer in the liquid-wall system, has a substantial effect on the thermostressed state of the pipe. This enables us to conclude that a complex investigation is needed for conjugate heat transfer in flows through pipes and channels and the thermostressed state of the walls of the washed constructions.

NOTATION

T, temperature; u_x, u_y, u_z, displacement vector components; T_{in}, temperature at the pipe inlet; T_w temperature of the pipe outer surface; λ, thermal conductivity coefficient; x, y, z, Cartesian coordinates; w_z, velocity profile for laminar stabilized flow in a triangular channel; $\bar{w}$, characteristic velocity; Pe, Peclet number; h, height of the isosceles triangle, comprising the contour of the channel cross section; λ_ℓ, a_ℓ, thermal conductivity and thermal diffusivity coefficients of the liquid; α_p, coefficient of linear expansion of the pipe material; l, length of the calculated portion of the pipe; Nu, Nusselt number; d_e, equivalent diameter ($d_e = 2/3 \cdot h$); $\sigma = \{\sigma_{XX}, \sigma_{YY}, \sigma_{ZZ}, \sigma_{XY}, \sigma_{XZ}, \sigma_{YZ}\}$, tensor of stresses in the dimensionless form; φ, half-angle at the vertex of the isosceles triangle; ν, Poisson coefficient for the pipe material.

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